MATHEMATICS – 4

(For Electrical, Electronics and Applied Electronics)

CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
MAT 204	PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS	BASIC SCIENCE COURSE	3	1	0	4

Preamble: This course introduces students to the modern theory of probability and statistics, covering important models of random variables and analysis of random processes using appropriate time and frequency domain tools. A brief course in numerical methods familiarises students with some basic numerical techniques for finding roots of equations, evaluating definite integrals solving systems of linear equations and solving ordinary differential equations which are especially useful when analytical solutions are hard to find.

Prerequisite: A basic course in one-variable and multi-variable calculus**.**

Course Outcomes: After the completion of the course the student will be able to

CO 1	Understand the concept, properties and important models of discrete random variables and, using them, analyse suitable random phenomena.
CO 2	Understand the concept, properties and important models of continuous random variables and, using them, analyse suitable random phenomena.
CO 3	Analyse random processes using autocorrelation, power spectrum and Poisson process model as appropriate.
CO 4	Compute roots of equations, evaluate definite integrals and perform interpolation on given numerical data using standard numerical techniques
CO 5	Apply standard numerical techniques for solving systems of equations, fitting curves on given numerical data and solving ordinary differential equations.

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	2	2	2	2					2		1
CO 2	3	2	2	2	2					2		1
CO 3	3	2	2	2	2					2		1
CO 4	3	2	2	2	2					2		1
CO 5	3	2	2	2	2					2		1

Assessment Pattern

Bloom's Category	Continuous Assessment Tests(%)		End Semester Examination(%)
	1	2	
Remember	10	10	10
Understand	30	30	30
Apply	30	30	30
Analyse	20	20	20
Evaluate	10	10	10
Create			

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Let X denote the number that shows up when an unfair die is tossed. Faces 1 to 5 of the die are equally likely, while face 6 is twice as likely as any other. Find the probability distribution, mean and variance of X.
2. An equipment consists of 5 components each of which may fail independently with probability 0.15. If the equipment is able to function properly when at least 3 of the components are operational, what is the probability that it functions properly?
3. X is a binomial random variable $B(n,p)$with $n = 100$and $p = 0.1$. How would you approximate it by a Poisson random variable?
4. Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X,Y)

Course Outcome 2 (CO2)

1. What can you say about $P(X = a)$for any real number $a$when $X$is (i) a discrete random variable? (ii) a continuous random variable?
2. A string, 1 meter long, is cut into two pieces at a random point between its ends. What is the probability that the length of one piece is at least twice the length of the other?

3. A random variable has a normal distribution with standard deviation 10. If the probability that it will take on a value less than 82.5 is 0.82, what is the probability that it will take on a value more than 58.3?

4. X and Y are independent random variables with X following an exponential distribution with parameter μ and Y following and exponential distribution with parameterλ. Find $P(X + Y \leqslant 1)$

Course Outcome 3(CO3):

1. A random process $X(t)$ is defined by $acos(\omega t + \Theta)$where a and ω are constants and Θ is uniformly distributed in$[0,2\pi]$. Show that $X(t)$ is WSS

2. How are the autocorrelation function and power spectral density of a WSS process are related to each other?

3. Find the power spectral density of the WSS random process $X(t)$, given the autocorrelation function $R_X(\tau) = 9e^{-|\tau|}$

4. A conversation in a wireless ad-hoc network is severely disturbed by interference signals according to a Poisson process of rate $\lambda = 0.01$ per minute. (a) What is the probability that no interference signals occur within the first two minutes of the conversation? (b) Given that the first two minutes are free of disturbing effects, what is the probability that in the next minute precisely 1 interfering signal disturbs the conversation? (c) Given that there was only 1 interfering signal in the first 3 minutes, what is the probability that there would be utmost 2 disturbances in the first 4 minutes?

Course Outcome 4(CO4):

1. Use Newton-Raphson method to find a real root of the equation $f(x) = e^{2x} - x - 6$correct to 4 decimal places.

2. Compare Newton's divided difference method and Lagrange's method of interpolation.

3. Use Newton's forward interpolation formula to compute the approximate values of the function $f$at $x = 0.25$from the following table of values of $x$and $f(x)$

x	0	0.5	1	1.5	2
f(x)	1.0000	1.0513	1.1052	1.1618	1.2214

4. Find a polynomial of degree 3 or less the graph of which passes through the points (-1, 3), (0,-4), (1,5) and (2,-6)

Course Outcome 5 (CO5):

1. Apply Gauss-Seidel method to solve the following system of equations
$$4x_1 - x_2 - x_3 = 3$$
$$-2x_1 + 6x_2 + x_3 = 9$$
$$-x_1 + x_2 + 7x_3 = -6$$

2. Using the method of least squares fit a straight line of the form $y = ax + b$to the following set of ordered pairs (x, y) :
(2,4), (3,5), (5,7), (7,10), (9,15)

3. Write the normal equations for fitting a curve of the form $y = a_0 + a_1x^2$to a given set of pairs of data points.

4. Use Runge-Kutta method of fourth order to compute $y(0.25)$and $y(0.5)$, given the initial value problem
$$y' = x + xy + y, y(0) = 1$$

Syllabus

Module 1 (Discrete probability distributions) 9 hours

(Text-1: *Relevant topics* from sections-3.1-3.4, 3.6, 5.1)

Discrete random variables and their probability distributions, Expectation, mean and variance, Binomial distribution, Poisson distribution, Poisson approximation to the binomial distribution, Discrete bivariate distributions, marginal distributions, Independent random variables, Expectation (multiple random variables)

Module 2 (Continuous probability distributions) 9 hours

(Text-1:*Relevant topics* from sections-4.1-4.4, 3.6, 5.1)

Continuous random variables and their probability distributions, Expectation, mean and variance, Uniform, exponential and normal distributions, Continuous bivariate distributions, marginal distributions, Independent random variables, Expectation (multiple random variables), i. i. d random variables and Central limit theorem (without proof).

Module 3 (Random Processes) 9 hours

(Text-2: *Relevant topics* from sections-8.1-8.5, 8.7, 10.5)

Random processes and classification, mean and autocorrelation, wide sense stationary (WSS) processes, autocorrelation and power spectral density of WSS processes and their properties, Poisson process-distribution of inter-arrival times, combination of independent Poisson processes(merging) and subdivision (splitting) of Poisson processes (**results without proof**).

Module 4 (Numerical methods -I) 9 hours

(Text 3- ***Relevant topics*** **from sections 19.1, 19.2, 19.3, 19.5)**

Errors in numerical computation-round-off, truncation and relative error, Solution of equations – Newton-Raphson method and Regula-Falsi method. Interpolation-finite differences, Newton's forward and backward difference method, Newton's divided difference method and Lagrange's method. Numerical integration-Trapezoidal rule and Simpson's 1/3rd rule (**Proof or derivation of the formulae not required for any of the methods in this module**)

Module 5 (Numerical methods -II) 9 hours

(Text 3- ***Relevant topics*** **from sections 20.3, 20.5, 21.1)**

Solution of linear systems-Gauss-Seidel and Jacobi iteration methods. Curve fitting-method of least squares, fitting straight lines and parabolas. Solution of ordinary differential equations-Euler and Classical Runge-Kutta method of second and fourth order, Adams-Moulton predictor-correction method (**Proof or derivation of the formulae not required for any of the methods in this module**)

Text Books

1. (Text-1) Jay L. Devore, *Probability and Statistics for Engineering and the Sciences*, 8th edition, Cengage, 2012
2. (Text-2) Oliver C. Ibe, *Fundamentals of Applied Probability and Random Processes,* Elsevier, 2005.
3. (Text-3) Erwin Kreyszig, *Advanced Engineering Mathematics*, 10 th Edition, John Wiley & Sons, 2016.

Reference Books

1. Hossein Pishro-Nik, *Introduction to Probability, Statistics and Random Processes*, Kappa Research, 2014 (Also available online at www.probabilitycourse.com)
2. V.Sundarapandian, *Probability, Statistics and Queueing theory*, PHI Learning, 2009
3. Gubner, *Probability and Random Processes for Electrical and Computer Engineers*, Cambridge University Press,2006.
4. B.S. Grewal, *Higher Engineering Mathematics*, Khanna Publishers, 36 Edition, 2010.

Assignments

Assignments should include specific problems highlighting the applications of the methods introduced in this course in physical sciences and engineering.

Course Contents and Lecture Schedule

No	Topic	No. of Lectures
1	**Discrete Probability distributions**	**9 hours**
1.1	Discrete random variables and probability distributions, expected value, mean and variance (discrete)	3
1.2	Binomial distribution-mean, variance, Poisson distribution-mean, variance, Poisson approximation to binomial	3
1.3	Discrete bivariate distributions, marginal distributions, Independence of random variables (discrete), Expected values	3
2	**Continuous Probability distributions**	**9 hours**
2.1	Continuous random variables and probability distributions, expected value, mean and variance (continuous)	2
2.2	Uniform, exponential and normal distributions, mean and variance of these distributions	4
2.3	Continuous bivariate distributions, marginal distributions, Independent random variables, Expected values, Central limit theorem.	3
3	**Random processes**	**9 hours**
3.1	Random process -definition and classification, mean , autocorrelation	2
3.2	WSS processes its autocorrelation function and properties	2
3.3	Power spectral density	2
3.4	Poisson process, inter-distribution of arrival time, merging and splitting	3
4	**Numerical methods-I**	**9 hours**
4.1	Roots of equations- Newton-Raphson, regulafalsi methods	2
4.2	Interpolation-finite differences, Newton's forward and backward formula,	3
4.3	Newton's divided difference method, Lagrange's method	2
4.3	Numerical integration-trapezoidal rule and Simpson's 1/3-rd rule	2
5	**Numerical methods-II**	**9 hours**
5.1	Solution of linear systems-Gauss-Siedal method, Jacobi iteration	2

	method	
5.2	Curve-fitting-fitting straight lines and parabolas to pairs of data points using method of least squares	2
5.3	Solution of ODE-Euler and Classical Runge-Kutta methods of second and fourth order	4
5.4	Adams-Moulton predictor-corrector method	1

Model Question Paper

(2019 Scheme)

Reg No: .... **Total Pages: 3**

Name: ....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION

(Month & year)

Course Code: MAT 204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

(For (i) Electrical and Electronics, (ii) Electronics and Communication, (iii) Applied Electronics and Instrumentation Engineering branches)

Max Marks :100 Duration : 3 Hours

PART A

(Answer *all* questions. Each question carries 3 marks)

1. Suppose X is binomial random variable with parameters $n = 100$ and $p = 0.02$. Find $P(X < 3)$ using Poisson approximation to X. (3)

2. The diameter of circular metallic discs produced by a machine is a random variable with mean 6cm and variance 2cm. Find the mean area of the discs. (3)

3. Find the mean and variance of the continuous random variable X with probability density function (3)
$$f(x) = \begin{cases} 2x - 4, & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

4. The random variable X is exponentially distributed with mean 3. Find $P(X > t + 3|X > t)$ where t is any positive real number. (3)

5. Give any two examples of a continuous time discrete state random processes. (3)

6. How will you calculate the mean, variance and total power of a WSS process from its autocorrelation function? (3)

7. Find all the first and second order forward and backward differences of y for the following set of (x, y) values: $(0.5, 1.13), (0.6, 1.19), (0.7, 1.26), (0.8, 1.34)$ (3)

8. The following table gives the values of a function $f(x)$ for certain values of x. (3)

x	0	0.25	0.50	0.75	1
$f(x)$	1	0.9412	0.8	0.64	0.5

Evaluate $\int_0^1 f(x)dx$ using trapezoidal rule.

9. Explain the principle of least squares for determining a line of best fit to a given data (3)

10. Given the initial value problem $y' = y + x, \quad y(0) = 0$, find $y(0.1)$ and $y(0.2)$ using Euler method. (3)

PART B

(Answer one question from each module)

MODULE 1

11. (a) The probability mass function of a discrete random variable is $p(x) = kx, x = 1, 2, 3$ where k is a positive constant. Find (i)the value of k (ii) $P(X \leq 2)$ (iii) $E[X]$ and (iv) $\text{var}(1 - X)$. (7)

    (b) Find the mean and variance of a binomial random variable (7)

OR

12. (a) Accidents occur at an intersection at a Poisson rate of 2 per day. what is the probability that there would be no accidents on a given day? What is the probability that in January there are at least 3 days (not necessarily consecutive) without any accidents? (7)

    (b) Two fair dice are rolled. Let X denote the number on the first die and $Y = 0$ or 1, according as the first die shows an even number or odd number. Find (i) the joint probability distribution of X and Y, (ii) the marginal distributions. (iii) Are X and Y independent ? (7)

MODULE 2

13. (a) The IQ of an individual randomly selected from a population is a normal distribution with mean 100 and standard deviation 15. Find the probability that an individual has IQ (i) above 140 (ii) between 120 and 130. (7)

    (b) A continuous random variable X is uniformly distributed with mean 1 and variance 4/3. Find $P(X < 0)$ (7)

OR

14. (a) The joint density function of random variables X and Y is given by (7)

$$f(x,y) = \begin{cases} e^{-(x+y)}, & x > 0, \quad y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

    Find $P(X + Y \leq 1)$. Are X and Y independent? Justify.

    (b) The lifetime of a certain type of electric bulb may be considered as an exponential random variable with mean 50 hours. Using central limit theorem, find the approximate probability that 100 of these electric bulbs will provide a total of more than 6000 hours of burning time. (7)

MODULE 3

15. (a) A random process $X(t)$ is defined by $X(t) = Y(t)\cos(\omega t + \Theta)$ where $Y(t)$ is a WSS process, ω is a constant and Θ is uniformly distributed in $[0, 2\pi]$ and is independent of $Y(t)$. Show that $X(t)$ is WSS (7)

    (b) Find the power spectral density of the random process $X(t) = a\sin(\omega_0 t + \Theta)$, ω_0 constant and Θ is uniformly distributed in $(0, 2\pi)$ (7)

OR

16. Cell-phone calls processed by a certain wireless base station arrive according to a Poisson process with an average of 12 per minute.

    (a) What is the probability that more than three calls arrive in an interval of length 20 seconds? (7)

    (b) What is the probability that more than 3 calls arrive in each of two consecutive intervals of length 20 seconds? (7)

MODULE 4

17\. (a) Use Newton-Raphson method to find a non-zero solution of $x = 2\sin x$. Start with $x_0 = 1$ (7)

(b) Using Lagrange's interpolating polynomial estimate $f(1.5)$ for the following data (7)

x	0	1	2	3
$y = f(x)$	0	0.9826	0.6299	0.5532

OR

18\. (a) Consider the data given in the following table (7)

x	0	0.5	1	1.5	2
$f(x)$	1.0000	1.0513	1.1052	1.1618	1.2214

Estimate the value of $f(1.80)$ using newton's backward interpolation formula.

(b) Evaluate $\int_0^1 e^{-x^2/2}dx$ using Simpson's one-third rule, dividing the interval $[0, 1]$ into 8 subintervals (7)

MODULE 5

19\. (a) Using Gauss-Seidel method, solve the following system of equations (7)

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

(b) The table below gives the estimated population of a country (in millions) for during 1980-1995 (7)

year	1980	1985	1990	1995
population	227	237	249	262

Plot a graph of this data and fit an appropriate curve to the data using the method of least squares. Hence predict the population for the year 2010.

OR

20\. (a) Use Runge-Kutta method of fourth order to find $y(0.2)$ given the initial value problem (7)

$$\frac{dy}{dx} = \frac{xy}{1 + x^2}, \quad y(0) = 1$$

Take step-size, $h = 0.1$.

(b) Solve the initial value problem (7)

$$\frac{dy}{dx} = x + y, \quad y(0) = 0,$$

in the interval $0 \le x \le 1$, taking step-size $h = 0.2$. Calculate $y(0.2), y(0.4)$ and $y(0.6)$ using Runge-Kutta second order method, and $y(0.8)$ and $y(1.0)$ using Adam-Moulton predictor-corrector method.

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